**Medgar Evers College – Mathematics**
## Calculus 1
**MTH 202, Fall 2017**
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## CAS Lab #4. Numerically estimating limits

### LIMITS

This very brief introduction to the limit of a function is excerpted from Section 2.2 of *OpenStax Calculus*[1] by G. Strang, E. Herman, et al.

The concept of a limit or limiting process, essential to the understanding of calculus, has been around for thousands of years. In fact, early mathematicians used a limiting process to obtain better and better approximations of areas of circles. Yet, the formal definition of a limit —as we know and understand it today— did not appear until the late 19th century.

The purpose of this Lab is to get a feeling for what the following definition means.

**Intuitive definition of limit.** Let $f(x)$ be a function defined at all values in an open interval containing $a$, with the possible exception of a itself, and let $L$ be a real number. If all values of the function $f(x)$ approach the real number $L$ as the values of $x(\neq a)$ approach the number $a$, then we say that the limit of $f(x)$ as $x$ approaches $a$ is $L$ and we write

$$\lim_{x\to a} f(x) = L.$$

**Precise mathematical definition of limit.** Let $f(x)$ be defined for all $x \neq a$ in an open interval containing $a$. Let $L$ be a real number. Then $\lim_{x\to a} f(x) = L$ if, for every $\varepsilon > 0$, there exists a $\delta > 0$, such that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

### NUMERICAL APPROACH TO LIMITS

The most direct way to estimate a limit $\lim_{x\to a} f(x)$ is to compute values of $f(x)$ as $x$ "approaches" $a$ and then to determine whether these values $f(x)$ seem to be approaching a single value $L$.

The following program computes values of

$$g(x) = \frac{x^3-1}{x^2-1}$$

as $x$ approaches $a = 1$ *from the right* (i.e. $x$ is greater than $a$).

```
g(x)=(x^3-1)/(x^2-1)
a=1
nsteps=7
for k in range(nsteps):
    x=(a+1/10^k).n()
    y=g(x).n(digits=20)
    print (x,y)
```

The command "`.n()`" tells Sage to use a decimal approximation, and the command "`.n(digits=20)`" tells Sage to use a digital approximation with 20 digits.

**Assignment.**
1. Copy the program above into a Sage Worksheet called "lab4" on https://cocalc.com and run it. Interpret the program's output.
2. Modify (a copy of) the program so that it computes values of $g(x)$ as $x$ approaches $a$ *from the left*. [Fact: *For* $\lim_{x\to a} f(x)$ *to exist,* $f(x)$ *must approach the same real number as x approaches a from the left and from the right.*]
3. Use (modified versions of) the program to solve problems 1, 2, 4, 5, 6 and 10 of WeBWorK Set 02.

[1] https://cnx.org/contents/i4nRcikn@2.66:dKCfyV9u@3/The-Limit-of-a-Function