

CAS Lab #5. The derivative

INSTANTANEOUS RATE OF CHANGE

Recall the definition of the *rate of change* or *derivative* of a function at a point from p.112 of the textbook [<http://www.math.smith.edu/~callahan/cic/calcl1.pdf>]. More formally, the **derivative** $f'(a)$ of the function $f(x)$ at $x = a$ is given by

$$f'(a) = \lim_{h \rightarrow a} \frac{f(a+h) - f(a)}{h}$$

The following *Python* program computes the value of the difference quotient

$$\frac{f(a+h) - f(a)}{h}$$

for $f(x) = 1/x$ and $a = 2$ for $h = .1, .01, \dots, .000001$:

```
f(x)=1/x
a=2
nsteps=6
for k in range(nsteps):
    h=(1/10^(k+1)).n()
    d=((f(a+h)-f(a))/h).n(digits=20)
    print (h,d)
```

ASSIGNMENT

- (1) Copy the program above into a Sage Worksheet called “lab5” on <https://cocalc.com> and run it. Based on the output, give your best estimate for $f'(2)$ when $f(x) = 1/x$ and say how many decimal places of accuracy it has.

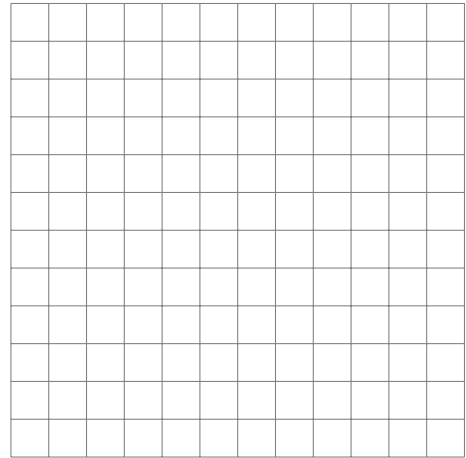
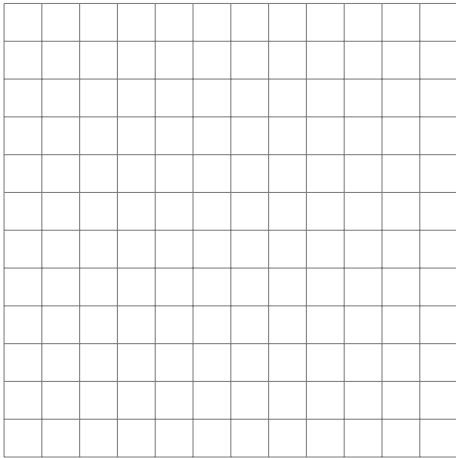
- (2) Use modified copies of the program above to give your best estimate for $f'(a)$ and say how many decimal places of accuracy it has, for the following choices of $f(x)$ and a :
 - (a) $f(x) = \sin(7x); a = 3$
 - (b) $f(x) = x^3; a = 200$
 - (c) $f(x) = 2^x; a = 5$

(3) Modify a copy of the program above to solve exercise 3 on page 131 of the textbook. [Write your answers to (b) and (c) in the space below]

(4) Solve exercises 2(a) and 2(d) on page 115 of the textbook using the online graphing tool <https://www.desmos.com/calculator> or <https://www.geogebra.org/graphing> [Use the space below to write down your successive estimates]

(5) Solve exercises 3(a) and 3(b) on page 115 of the textbook using your online graphing tool.

(6) Solve exercises 6(a) and 6(b) on page 116 of the textbook on the grids below



(7) Use the space below (and additional paper if necessary) to answer questions (a) through (e) of exercise 7 on page 117 of the textbook