**Medgar Evers College – Mathematics**

# Calculus 1

**MTH 202, Fall 2017**

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## CAS Lab #6. Newton's Method

Will be collected on **Monday, December 4**, and graded

### Finding roots

"Newton's method" is a technique for finding successive approximations of solutions to an equation of the form

$$g(x) = 0$$

where $g(x)$ is a differentiable function. It is described in the section "Local Linearity and the Tangent Line" on pages 320-322 of the textbook[1].

### The algorithm

The following *Python* program, which is an adaptation of the program on page 323 of the textbook, is an implementation of Newton's method: it finds successive approximations of the root of

$$g(x) = 4x^3 + 3x^2 + 2x + 1$$

```
start = -1
numberofsteps = 6
numberofdigits = 11
x = start.n(digits=numberofdigits)
for n in range(numberofsteps):
    print n, x
    g = (4*x^3 + 3*x^2 + 2*x + 1).n(digits=numberofdigits)
    gprime = (12*x^2 + 6*x + 2).n(digits=numberofdigits)
    x = x - g/gprime
```

### Assignment

1. After reading the section on "Local Linearity and the Tangent Line" on pages 320-322 of the textbook, fill in the blanks in the following paragraph.

   Starting from an initial *guess* $x_0$ of a solution of $g(x) = 0$, Newton's method computes the next estimate $x_1$ as the $x$-intercept of the ____________ line to the graph of $g(x)$ at $x =$ _____. In the case where $g(x) = 4x^3 + 3x^2 + 2x + 1$ and $x_0 = -1$ we therefore find $x_1$ as follows. The derivative $g'(x) =$ ____________ and consequently $g'(x_0) = g'($_____$) =$ _____. This last number is the ____________ of the tangent line to the graph of $g(x)$ at $x =$ _____. Since $g(x_0) = g($_____$) =$ _____, the point-slope form of the equation of the tangent line to the graph of $g(x)$ at $x = x_0$ is $y -$ _____ $=$ _____$(x -$ _____$)$ and consequently its slope-intercept form is $y =$ _______. Consequently, $x_1$ is the solution of the equation _________ $= 0$, and so $x_1 =$ _______.

[1] http://math.smith.edu/~callahan/cic/calc1.pdf

2. Let $g(x) = x^2 - 5$. Use Newton's method with starting point $x_0 = 2$ to compute the next estimate $x_1$ for the solution of $x^2 - 5 = 0$ near $x = 2$ *by hand* (you may use a calculator to do arithmetic). Show your work.

3. We continue with $g(x) = x^2 - 5$. Apply Newton's method, with starting point $x_1$, which you found in the previous question, to compute the next estimate $x_2$ of the root *by hand* (you may use a calculator to do arithmetic). Show your work.

4. Copy the program above into a Sage Worksheet called "lab6" on `https://cocalc.com` and run it. Copy the output in the space below. In the last line of output, underline all the digits that have stabilized.

5. Let $r$ be the real solution to the equation $4x^3 + 3x^2 + 2x + 1 = 0$. Based on the stabilized digits above, we know that $r =$ ________________...

6. Modify a copy of the program above to find the first 15 decimals of the root $r$ of $4x^3 + 3x^2 + 2x + 1 = 0$. What is the smallest value of $n$ (as in the program) for which (at least) 15 decimals of $r$ have stabilized? $n =$ _____

7. Modify a copy of the program above to find the first the first 20 digits of $\sqrt{5}$. (Note that $\sqrt{5}$ is a solution to $x^2 - 5 = 0$). Print and attach a copy of the program and its output. Underline the first 20 digits of $\sqrt{5}$ in the first output line where you know they are correct, based on our "stabilization method".