

$$\begin{aligned} \text{II (b)} \quad y' &= \left(\frac{1}{6}x^6 + \frac{1}{5}x^5 + \frac{1}{4}x^4 \right)' \\ &= \left(\frac{1}{6}x^6 \right)' + \left(\frac{1}{5}x^5 \right)' + \left(\frac{1}{4}x^4 \right)' \\ &= \frac{1}{6}(x^6)' + \frac{1}{5}(x^5)' + \frac{1}{4}(x^4)' \\ &= \frac{1}{6}(6x^5) + \frac{1}{5}(5x^4) + \frac{1}{4}(4x^3) \end{aligned}$$

[by the sums rule]

[by the constant multiple rule, 3 times]

[by the power rule, 3 times]

answer: $y' = x^5 + x^4 + x^3$

$$\begin{aligned} \text{(c)} \quad y' &= \left(-2x^2 \ln(x) \right)' \\ &= -2 \left(x^2 \ln(x) \right)' \\ &= -2 \left[(x^2)' \ln(x) + x^2 (\ln(x))' \right] \\ &= -2 \left[2x \ln(x) + x^2 \left(\frac{1}{x} \right) \right] \end{aligned}$$

[by the constant multiple rule]

[by the product rule]

[by the power rule and "ln-rule"]

answer: $y' = -4x \ln(x) - 2x$

$$\begin{aligned} \text{(d)} \quad y' &= \left(\sqrt{x^4 - 3x} \right)' \\ &= \frac{1}{2\sqrt{x^4 - 3x}} (x^4 - 3x)' \\ &= \frac{1}{2\sqrt{x^4 - 3x}} (4x^3 - 3) \end{aligned}$$

chain rule
 $y = \sqrt{u}$ with $u = x^4 - 3x$
 $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$ by the power rule

[by the sums rule, the constant multiple rule and twice the power rule]

answer: $y' = \frac{4x^3 - 3}{2\sqrt{x^4 - 3x}}$

$$\begin{aligned}
 (e) \quad y' &= (3e^{-x} \sin(2\pi x))' \\
 &= 3(e^{-x} \sin(2\pi x))' \quad [\text{by the constant multiple rule}] \\
 &= 3[(e^{-x})' \sin(2\pi x) + e^{-x} (\sin(2\pi x))'] \quad [\text{by the product rule}]
 \end{aligned}$$

$$\begin{aligned}
 (e^{-x})' &= e^{-x} \cdot (-x)' \quad [\text{by the chain rule and the exponential rule}] \\
 &= e^{-x} (-1) \quad [\text{by the constant multiple rule and the power rule}]
 \end{aligned}$$

$$\begin{aligned}
 (\sin(2\pi x))' &= \cos(2\pi x) \cdot (2\pi x)' \quad [\text{by the chain rule and the sine rule}] \\
 &= \cos(2\pi x) \cdot 2\pi \quad [\text{by the constant multiple rule and the power rule}]
 \end{aligned}$$

$$y' = 3[-e^{-x} \sin(2\pi x) + e^{-x} 2\pi \cos(2\pi x)]$$

$$\text{answer: } y' = 3e^{-x}(-\sin(2\pi x) + 2\pi \cos(2\pi x))$$

5 from the graph: $F(-2) = 1$ $F'(-2) = 1$
 $G(-2) = 4$ $G'(-2) = -1$

$$\begin{aligned}
 P'(-2) &= F'(-2)G(-2) + F(-2)G'(-2) \quad [\text{product rule}] \\
 &= (1)(4) + (1)(-1) \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 Q(-2) &= \frac{F'(-2)G(-2) - F(-2)G'(-2)}{G(-2)^2} \quad [\text{quotient rule}] \\
 &= \frac{(1)(4) - (1)(-1)}{(4)^2} = \frac{5}{16}
 \end{aligned}$$

4 a) f is increasing when f' is positive. This is the case for x in $(-3, 1) \cup (2, \infty)$

f is decreasing when f' is negative. This is the case for x in $(-\infty, -3) \cup (1, 2)$

b) f has a local maximum at $x = 1$.

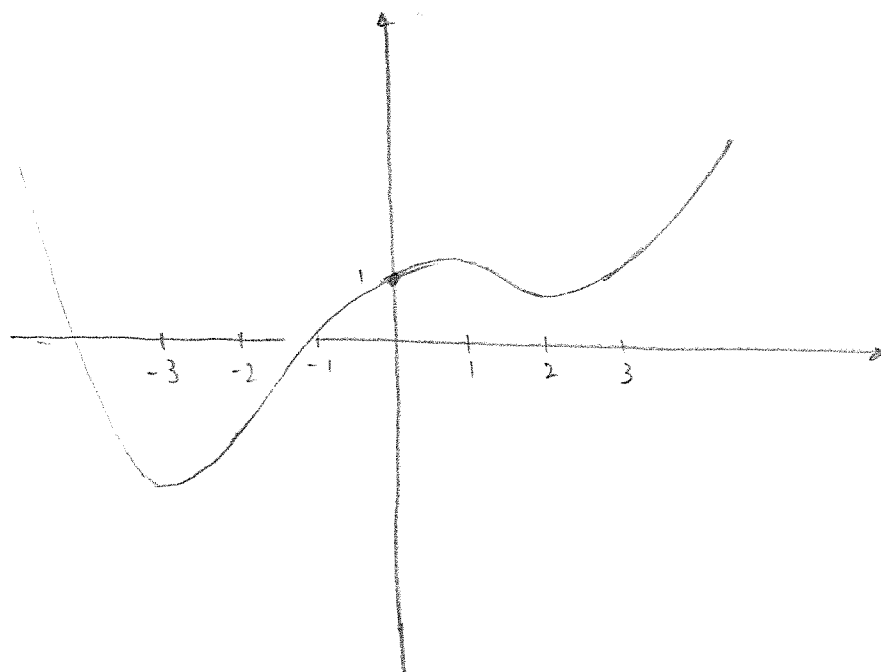
f has a local minimum at $x = -3$ and at $x = 2$

c) f is concave upwards when f' is increasing. This is the case for x in $(-\infty, -1.5) \cup (1.5, \infty)$

f is concave downward when f' is decreasing. This is the case for x in $(-1.5, 1.5)$

d) f has inflection points at $x = -1.5$ and at $x = 1.5$

e)



2. (You do not need to copy this problem. You can just fill in the blanks) On each line (_____) write the name of the differentiation rule that justifies that step of the calculation. (10)

$$f(x) = x^3 + (x^2 + 3)^4 + \sqrt{8}$$

$$f'(x) = (x^3 + (x^2 + 3)^4 + \sqrt{8})'$$

$$= (x^3)' + ((x^2 + 3)^4)' + (\sqrt{8})'$$

by the sums rule

$$= (x^3)' + ((x^2 + 3)^4)' + 0$$

by the constant rule

$$= (x^3)' + 4(x^2 + 3)^3(x^2 + 3)'$$

by the chain rule

$$= (x^3)' + 4(x^2 + 3)^3[(x^2)' + (3)']$$

by the sums rule

$$= (x^3)' + 4(x^2 + 3)^3(x^2)'$$

by the constant rule

$$= 3x^2 + 4(x^2 + 3)^3 2x$$

by the power rule

$$= 3x^2 + 8x(x^2 + 3)^3$$

3. Neatly copy the solution to the next question and correctly fill in the blanks according to the context. You must copy the entire solution to receive credit. Answers that just list responses will not be graded. Highlight each response you entered in a blank space using a highlighter. (12)

Find the equation of the tangent line to the graph of the function $y = x^2 - 5$ at the point with x -coordinate 2.

Solution:

The point on the graph of $y = x^2 - 5$ with x -coordinate 2 is (2, -1). The slope of the tangent line at $x = 2$ is $y'(2)$. We first compute $y' = \underline{2x}$ and then $y'(2) = \underline{4}$. So, the desired tangent line is the line through the point (2, -1) and with slope 4. Therefore, the point-slope formula tells us that its equation is $y - \underline{(-1)} = \underline{4}(x - \underline{2})$, or, in slope-intercept form, $y = \underline{4x - 9}$.

4. Problem 5 on the Practice Midterm Exam¹ (10)
5. Problem 8 on the Practice Midterm Exam (10)

¹available from <http://bvans.net/mth202>