# WA 3

[1] **Proposition:** If $r$ is a real number such that $r^3 = 2$, then $r$ is an irrational number.

**Proof:** We will prove the proposition by contradiction. Therefore, we assume that the proposition is false. That is, we assume that $r$ is a rational number such that $r^3 = 2$.

As $r$ is rational, there exist integers $k$ and $l$ such that

(a) $l \neq 0$

(b) $r = \frac{k}{l}$

(c) $k$ and $l$ have no common factor greater than 1

Taking the third power of the equation in (b) and using that $r^3 = 2$ we obtain

$$2 = \left(\frac{k}{l}\right)^3 = \frac{k^3}{l^3} ,$$

which implies that $k^3 = 2l^3$. Since $l^3 \in \mathbb{Z}$, this implies that $k^3$ is even. By exercise 1(b) of section 3.2 we obtain that $k$ is even and so there exists $q \in \mathbb{Z}$ such that $k = 2q$. Substituting this into the equation $k^3 = 2l^3$ we find

$$\begin{aligned} 2l^3 &= (2q)^3 \\ &= 8q^3 \end{aligned}$$

Dividing by 2 gives us that $l^3 = 2(2q^3)$ and therefore

that $l^3$ is even. Using exercise 1(b) of Section 3.2 once more we deduce that $l$ is even.
We have deduced that both $l$ and $k$ are even, which contradicts (b). This contradiction shows that the proposition cannot be false and is therefore true. ∎

$\boxed{2}$ (a) This proposition is false. Here is a counterexample: $m = 0$, $x = \sqrt{2}$. This is a counterexample because $m=0$ is an integer, $x = \sqrt{2}$ is irrational and $m \cdot x = 0$ is rational.
The error in the proof is in the sentence "Hence, we may conclude that $mx \neq \frac{ma}{b}$": when $m = 0$ this is not true.

(b) This proposition is true and the proof is correct.