P Set 2

$\boxed{1}$

$$\left[\begin{array}{cccc|c} 3 & -6 & -1 & 1 & 7 \\ -1 & 2 & 2 & 3 & 1 \\ 4 & -8 & -3 & -2 & 6 \end{array}\right] \underset{\sim}{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} -1 & 2 & 2 & 3 & 1 \\ 3 & -6 & -1 & 1 & 7 \\ 4 & -8 & -3 & -2 & 6 \end{array}\right]$$

$$\underset{4R_1+R_2}{\overset{3R_1+R_2}{\sim}} \left[\begin{array}{cccc|c} -1 & 2 & 2 & 3 & 1 \\ 0 & 0 & 5 & 10 & 10 \\ 0 & 0 & 5 & 10 & 10 \end{array}\right] \underset{\sim}{-R_2+R_3} \left[\begin{array}{cccc|c} -1 & 2 & 2 & 3 & 1 \\ 0 & 0 & 5 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$5x_3 + 10x_4 = 10 \implies x_3 = 2 - 2x_4$$

$$-x_1 + 2x_2 + 2x_3 + 3x_4 = 1 \implies x_1 = -1 + 2x_2 + 2x_3 + 3x_4$$

$$= -1 + 2x_2 + 2(2 - 2x_4) + 3x_4$$

$$x_1 = 3 + 2x_2 - x_4$$

$$\text{solution set} = \left\{ \begin{pmatrix} 3+2s-t \\ s \\ 2-2t \\ t \end{pmatrix} : s, t \in \mathbb{R} \right\}$$

or $$\text{solution set} = \left\{ \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ -2 \\ 1 \end{pmatrix} : s, t \in \mathbb{R} \right\}$$

or $$\text{solution set} = \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \end{pmatrix} + \text{span}\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \\ 1 \end{pmatrix} \right\}$$

$\boxed{3}$ Continuing from the last augmented matrix in $\boxed{1}$

$$\left[\begin{array}{cccc|c} -1 & 2 & 2 & 3 & 1 \\ 0 & 0 & 5 & 10 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \underset{(\frac{1}{5})R_2}{\overset{(-1)R_1}{\sim}} \left[\begin{array}{cccc|c} 1 & -2 & -2 & -3 & -1 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\overset{2R_2+R_1}{\sim} \left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & -3 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 3 + 2x_2 - x_4 \\ x_3 = 2 \quad\quad -2x_4 \end{array}$$

and we obtain same solution set as in $\boxed{1}$

$\boxed{2}$ Ex 2.2.4

$$\left[\begin{array}{cc|c} 1 & -2 & b_1 \\ 2 & -4 & b_2 \\ -6 & 12 & b_3 \end{array}\right] \underset{6R_1+R_3}{\overset{-2R_1+R_2}{\sim}} \left[\begin{array}{cc|c} 1 & -2 & b_1 \\ 0 & 0 & -2b_1+b_2 \\ 0 & 0 & 6b_1+b_3 \end{array}\right]$$

$A\underline{x} = \underline{b}$ is consistent $\iff -2b_1+b_2=0$ and $6b_1+b_3=0$

$\iff b_2 = 2b_1$ and $b_3 = -6b_1$

Ex 2.2.6

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & b_1 \\ -2 & 3 & -1 & b_2 \\ 3 & -3 & 0 & b_3 \\ 2 & 0 & -2 & b_4 \end{array}\right] \underset{-2R_1+R_4}{\overset{\substack{2R_1+R_2 \\ -3R_1+R_3}}{\sim}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & b_1 \\ 0 & 3 & -3 & 2b_1+b_2 \\ 0 & -3 & 3 & -3b_1+b_3 \\ 0 & 0 & 0 & -2b_1+b_4 \end{array}\right] \overset{R_2+R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -1 & b_1 \\ 0 & 3 & -3 & 2b_1+b_2 \\ 0 & 0 & 0 & -b_1+b_2+b_3 \\ 0 & 0 & 0 & -2b_1+b_4 \end{array}\right]$$

$A\underline{x} = \underline{b}$ is consistent $\iff -b_1+b_2+b_3=0$ and $-2b_1+b_4=0$

$\boxed{4}$ Take, for example, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix}$

then $A \cdot B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A \cdot C$