

Linear Algebra II

MTH 317, Section 001, Spring 2018

Instructor: Bart Van Steirteghem

Problem Set 4. Due on Wednesday February 28 at the start of class.

Staple your solutions to the following problems behind this sheet.

Collaboration on homework is encouraged; individually written solutions are required.

At the start of each problem, list the sources other than the textbook that you consulted to solve the problem, and all the people you discussed the problem with. For example:

Sources:

(i) G. Schay (2012). *A concise introduction to linear algebra*. New York, NY: Springer.

(ii) MathDoctorBob video “Examples of Linear Maps”

Discussed with: Jane Smith (classmate), John Doe (tutor), Prof. Van Steirteghem, Prof. Holder

Problems

1. If A is an $n \times n$ matrix, how are $\det(A)$ and $\det(3A)$ related? (Hint: $\det(3A) = 3 \det(A)$ is only true when $n = 1$.)

2. How are the determinants of A and B related if

(a) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and $B = \begin{pmatrix} 3a_{11} & 5a_{12} & 7a_{13} \\ 3a_{21} & 5a_{22} & 7a_{23} \\ 3a_{31} & 5a_{32} & 7a_{33} \end{pmatrix}$?

(b) $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ and $B = \begin{pmatrix} 3a_{11} & 4a_{12} + 5a_{11} & 5a_{13} \\ 3a_{21} & 4a_{22} + 5a_{21} & 5a_{23} \\ 3a_{31} & 4a_{32} + 5a_{31} & 5a_{33} \end{pmatrix}$?

3. Prove that

$$\det \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} = (c - a)(c - b)(b - a)$$

4. Find a “nice” expression in terms of a_0, a_1, \dots, a_{n-1} and t for $\det(A + tI_n)$ where

$$A = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 & a_0 \\ -1 & 0 & 0 & \dots & 0 & a_1 \\ 0 & -1 & 0 & \dots & 0 & a_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & a_{n-2} \\ 0 & 0 & 0 & \dots & -1 & a_{n-1} \end{pmatrix}$$

(Hint: row expansion and induction is probably the best strategy)