Medgar Evers College – Mathematics

Linear Algebra II

MTH 317, Section 001, Spring 2018

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Problem Set 5.

Collaboration on homework is encouraged; individually written solutions are required.

1. Show that the following two subspaces U and V of $\mathbb{R}^3$ are equal:
$$U = \langle (1,2,3),(3,2,1) \rangle, \quad V = \langle (4,5,6),(6,5,4) \rangle.$$
2. Find a basis of $U \cap V$, where U and V are the following two subspaces of $\mathbb{R}^3$:
$$U = \langle (1,2,3),(1,1,1) \rangle, \quad V = \langle (1,5,6),(1,0,1) \rangle.$$
3. Let V be the vector space of (real) polynomials in the variable x of degree at most 2. Consider the following subset of V:
$$U := \{p \in V \colon p(x) = p(1-x)\}.$$
   (a) Prove that U is a subspace of V.
   (b) Find a basis of U.
   (c) Complete the basis you found in part (3b) to a basis of V.
4. Let W_1 and W_2 be subspaces of $\mathbb{R}^5$ with $\dim W_1 = \dim W_2 = 3$. What dimension can $W_1 \cap W_2$ have? Illustrate each possible value of $\dim(W_1 \cap W_2)$ with an example.
5. Let V be the vector space of (real) polynomials in the variable x of degree at most 3. Let U be the subspace of V spanned by
$$p_1 = x^3 - x^2, \quad p_2 = x^3 - x, \quad p_3 = x^2 - x, \quad \text{and} \quad p_4 = x^3 - 1.$$
   (a) Prove that U is the set of all polynomials $p \in V$ which have a zero at $x = 1$.
   (b) Find a subset of $\{p_1, p_2, p_3, p_4\}$ which is a basis of U, and complete it to a basis of V.
6. Let v and w be linearly independent vectors in a vector space V and let $\alpha, \beta \in \mathbb{R}$. Prove: the vectors $x = \alpha v + \beta w$ and $y = \beta v + \alpha w$ are linearly dependent if and only if $\alpha = \beta$ or $\alpha = -\beta$.