**Medgar Evers College – Mathematics**

# Linear Algebra II

**MTH 317, Section 001, Spring 2018**

Instructor: Bart Van Steirteghem

## **Problem Set 6.** Due on Wednesday March 21 at the start of class.

Staple your solutions to the following problems behind this sheet.
Collaboration on homework is encouraged; individually written solutions are required.
At the start of each problem, list the sources other than the textbook that you consulted to solve the problem, and all the people you discussed the problem with. For example:

Sources:
(i) G. Schay (2012). *A concise introduction to linear algebra*. New York, NY: Springer.
(ii) MathDoctorBob video "Examples of Linear Maps"

Discussed with: Jane Smith (classmate), John Doe (tutor), Prof. Van Steirteghem, Prof. Holder

If you did not consult any sources, or did not discuss the problem with anybody, then state this (for example: "Discussed with nobody.")

**Problems**

1. Find all $\alpha \in \mathbb{R}$ for which the matrix
$$\begin{pmatrix} \alpha & \alpha & \alpha \\ \alpha & 1 & 1 \\ \alpha & 1 & \alpha \end{pmatrix}$$
is invertible, and compute its inverse when it is invertible.

2. Consider the following matrices, which depend on the parameter $s \in \mathbb{R}$:
$$A = \begin{pmatrix} -s & s & -1 \\ 0 & -1 & s \\ s^2 & 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 & s \\ 0 & 1-s & 0 \\ s & s & 1 \end{pmatrix}$$
(a) For which values of $s$ is $A$ invertible?
(b) For which values of $s$ is $B$ invertible?
(c) Compute the rank of $A \cdot B$, as a function of $s$.

3. Consider the following three vectors in $\mathbb{R}^3$:
$$b_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$
and the following two vectors in $\mathbb{R}^2$:
$$c_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad c_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

(a) Show that $b_1, b_2, b_3$ is a basis of $\mathbb{R}^3$ and that $c_1, c_2$ is a basis of $\mathbb{R}^2$.

(b) Let $\varphi\colon \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation that, with respect to the canonical bases of $\mathbb{R}^3$ and $\mathbb{R}^2$, is given by the matrix

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & 1 \end{pmatrix}.$$

Determine the matrix of $\varphi$ with respect to bases in part (3a).

4. Let $V$ be the space of $2 \times 2$ matrices and consider the following two linear transformations

$$\varphi_1\colon V \to V, A \mapsto A + A^{tr};$$
$$\varphi_2\colon V \to V, A \mapsto A \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Compute the matrices of $\varphi_1$ and $\varphi_2$ with respect to the following basis of $V$:

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$