

Linear Algebra II

MTH 317, Section 001, Spring 2018

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Problem Set 7.

Collaboration on homework is encouraged; individually written solutions are required.

Problems

1. Let \mathcal{P}_1 be the space of linear polynomials in the variable x . Find the change of coordinates matrix that changes the coordinates in the basis $1, 1 + x$ in \mathcal{P}_1 to the coordinates in the basis $1 - x, 2x$.
2. Consider the system of vectors

$$\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 3 \\ 2 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

- (a) Prove that this system is a basis of \mathbb{R}^4 . (Try to do a minimal amount of computations)
 - (b) Find the change of coordinate matrix that changes the coordinates in this basis to the standard coordinates in \mathbb{R}^4 . In other words, if F is the basis above, and E is the standard basis of \mathbb{R}^4 , compute $\text{Mat}_{EF}(\text{Id})$.
3. For $d \in \mathbb{N}$, let \mathcal{P}_d be the vector space of real polynomials in the variable x of degree at most d . Consider the linear transformation

$$R: \mathcal{P}_d \rightarrow \mathcal{P}_d, R(p) = p'$$

- (a) Prove that \mathcal{P}_k , with $k \leq d$ is an invariant subspace of \mathcal{P}_d (with respect to R).
- (b) Prove that the only invariant subspaces (with respect to R) are $\{0\}$ and \mathcal{P}_k , where $k \in \{0, 1, 2, \dots, d\}$.