

Problem Set 9.

Collaboration on homework is encouraged; individually written solutions are required.

Assignment. For each of the terms below:

- write down the definition of the term
- illustrate the definition by at least one example
- state one theorem involving the term (try to choose a theorem that you think is important)
- illustrate the theorem by at least one example

You may use definitions and theorems from the course textbook, or from another source. Give a precise reference for each definition and theorem, for example

Definition 3.1 on page 81 of [Shafarevich, I. & Remizov, A. (2013). *Linear algebra and geometry*. New York, NY: Springer.]

or

Definition 2.1 on page 6 of [Treil, S. (2017). *Linear Algebra Done Wrong*.]

Terms to define.

1. vector space
2. subspace of a vector space
3. sum of two subspaces and direct sum of two subspaces
4. linearly independent collection of vectors
5. span of a collection of vectors
6. basis of a vector space
7. dimension of a vector space
8. linear transformation (or linear map) between two vector spaces
9. matrix of a linear transformation with respect to a choice of bases
10. kernel (or nullspace) of a linear transformation
11. image (or range or column space) of a linear transformation
12. rank of a matrix
13. invertible matrix
14. isomorphism of vector spaces, invertible linear transformation
15. eigenvalue and eigenvector
16. characteristic polynomial of a matrix and of a linear transformation
17. algebraic multiplicity of an eigenvalue
18. geometric multiplicity of an eigenvalue